UAF Tsunami Code

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1. Model equations

The numerical model is based on the vertically integrated nonlinear shallow water equations of motion and continuity with friction and Coriolis force. Written in a spherical coordinate system, they are:

$$\frac{\partial U}{\partial t}+\frac{U}{R\cos\varphi}\frac{\partial U}{\partial\lambda}+\frac{V}{R}\frac{\partial U}{\partial\varphi}-fV=-\frac{g}{R\cos\varphi}\frac{\partial\xi}{\partial\lambda}-\frac{rUW}{D} \tag{1}$$

$$\frac{\partial V}{\partial t}+\frac{U}{R\cos\varphi}\frac{\partial V}{\partial\lambda}+\frac{V}{R}\frac{\partial V}{\partial\varphi}+fU=-\frac{g}{R}\frac{\partial\xi}{\partial\varphi}-\frac{rVW}{D} \tag{2}$$

$$\frac{\partial\xi}{\partial t}=\frac{\partial\eta}{\partial t}-\frac{1}{R\cos\varphi}\left[\frac{\partial(DU)}{\partial\lambda}+\cos\varphi\frac{\partial(DV)}{\partial\varphi}\right], \tag{3}$$

where λ is longitude, φ is latitude, t is time, U and V are horizontal velocity components along longitude and latitude, $W=\sqrt{U^2+V^2}$, ξ is variation of sea level from equilibrium, η is the bottom displacement, g is the gravity acceleration, R is radius of the Earth, f is the Coriolis parameter, $D=(H+\xi-\eta)$ is the total water depth, and r is the bottom friction coefficient (wse use a value of 0.033).

2. Numerical scheme

An explicit in time finite-difference scheme is used to solve equations (1)-(3). This scheme solves for sea level heights and velocities by applying a space-staggered grid (Fig. 1). In finite-difference form equations (1)-(3) become:

$$U^{m}_{j,k}=U^{m-1}_{j,k}-\frac{T}{R\cos\varphi_k\Delta\lambda}(UPOS(U^{m-1}_{j,k}-U^{m-1}_{j-1,k})+UNEG(U^{m-1}_{j+1,k}-U^{m-1}_{j,k}))-$$
$$\frac{T}{R\Delta\varphi}\left(VAUP(U^{m-1}_{j,k}-U^{m-1}_{j,k-1})+VAUN(U^{m-1}_{j,k+1}-U^{m-1}_{j,k})\right)+2T\omega\sin\varphi_k VAU-$$
$$\frac{gT}{R\cos\varphi_k\Delta\lambda}\left(\xi^{m-1}_{j,k}-\xi^{m-1}_{j-1,k}\right)-\frac{rT}{H_1}U^{m-1}_{j,k}\left[\left(U^{m-1}_{j,k}\right)^2+\left(VAU\right)^2\right]^{\frac{1}{2}}$$

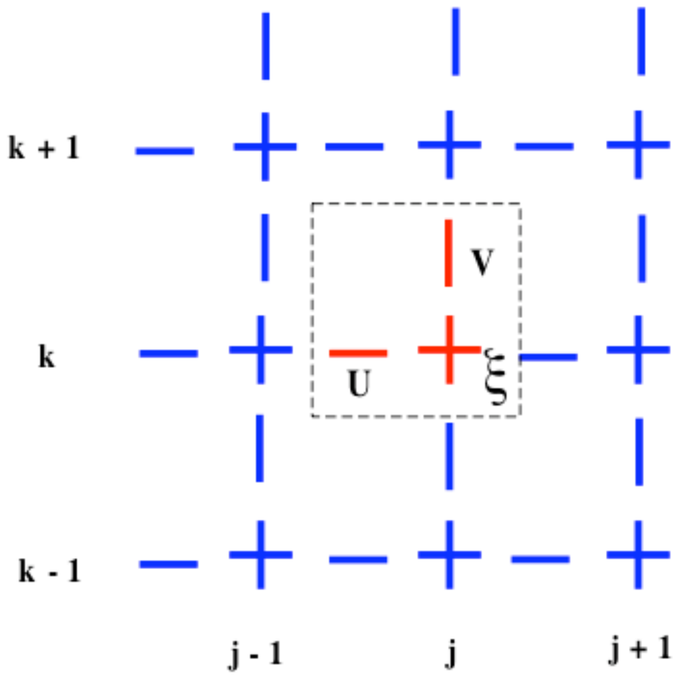

Figure 1. Space-stagerd grid for solving finite-difference equations.

$$V^{m}_{j,k} = V^{m-1}_{j,k} - \frac{T}{R\cos\varphi_k \Delta\lambda}(UAVP(V^{m-1}_{j,k} - V^{m-1}_{j-1,k}) + UAVN(V^{m-1}_{j+1,k} - V^{m-1}_{j,k})) -$$
$$\frac{T}{R\Delta\varphi}\left(VPOS(V^{m-1}_{j,k} - V^{m-1}_{j,k-1}) + VNEG(V^{m-1}_{j,k+1} - V^{m-1}_{j,k})\right) - 2T\omega UAV\left(\sin\varphi_k + \sin\varphi_{k+1}\right)/2 -$$
$$\frac{gT}{R\Delta\varphi}\left(\xi^{m-1}_{j,k+1} - \xi^{m-1}_{j,k}\right) - \frac{rT}{H_2}V^{m-1}_{j,k}\left[\left(V^{m-1}_{j,k}\right)^2 + \left(UAV\right)^2\right]^{1/2}$$

$$\xi^{m}_{j,k} = \xi^{m-1}_{j.k} - \frac{T}{R\cos\varphi_k}\left(\frac{U^{m}_{j+1,k}B_1 - U^{m}_{j,k}B_2}{\Delta\lambda} + \frac{S_1V^{m}_{j,k}B_3 - S_2V^{m}_{j,k-1}B_4}{\Delta\varphi}\right)$$

$$UPOS = 0.5\left(U^{m-1}_{j,k} + \left|U^{m-1}_{j,k}\right|\right)$$
$$UNEG = 0.5\left(U^{m-1}_{j,k} - \left|U^{m-1}_{j,k}\right|\right)$$
$$VAU = 0.25\left(V^{m-1}_{j,k} + V^{m-1}_{j-1,k} + V^{m-1}_{j,k-1} + V^{m-1}_{j-1,k-1}\right)$$
$$VAUP = 0.5\left(VAU + \left|VAU\right|\right)$$
$$VAUN = 0.5\left(VAU - \left|VAU\right|\right)$$
$$H_1 = 0.5\left(D^{m-1}_{j,k} + D^{m-1}_{j-1,k}\right)$$

$$VPOS = 0.5\left(V_{j,k}^{m-1} + \left|V_{j,k}^{m-1}\right|\right)$$
$$VNEG = 0.5\left(V_{j,k}^{m-1} - \left|V_{j,k}^{m-1}\right|\right)$$
$$UAV = 0.25\left(U_{j,k}^{m-1} + U_{j+1,k}^{m-1} + U_{j,k+1}^{m-1} + U_{j+1,k+1}^{m-1}\right)$$
$$UAVP = 0.5\left(UAV + \left|UAV\right|\right)$$
$$UAVN = 0.5\left(UAV - \left|UAV\right|\right)$$
$$H_2 = 0.5\left(D_{j,k}^{m-1} + D_{j+1,k}^{m-1}\right)$$

$$B_1 = 0.5\left(D_{j,k}^{m-1} + D_{j+1,k}^{m-1}\right)$$
$$B_2 = 0.5\left(D_{j,k}^{m-1} + D_{j-1,k}^{m-1}\right)$$
$$B_3 = 0.5\left(D_{j,k}^{m-1} + D_{j+1,k}^{m-1}\right)$$
$$B_4 = 0.5\left(D_{j,k}^{m-1} + D_{j+1,k}^{m-1}\right)$$
$$S_1 = 0.5\left(\cos\varphi_k + \cos\varphi_{k+1}\right)$$
$$S_2 = 0.5\left(\cos\varphi_k + \cos\varphi_{k-1}\right)$$

T – time step

$R\Delta\varphi$ - North-south space step

$R\cos\varphi\Delta\lambda$ - East-west space step

3. Stability condition

The time step T and space step h should satisfy the CFL condition:

$$T \le \frac{h}{\sqrt{2gH_{\max}}} ,$$

where h is the minimum space step, and H_{max} is the maximum water depth in the domain.

4. Boundary conditions

At the water-land boundary (shoreline), the velocity component normal to the boundary is set to zero. The radiation conditions are taken at the open ocean boundaries:

$$U_{j,k}^{m} = \pm\xi_{j,k}^{m-1}\sqrt{\frac{g}{D_{j,k}}}$$
$$V_{j,k}^{m} = \pm\xi_{j,k}^{m-1}\sqrt{\frac{g}{D_{j,k}}}$$

5. Nested grids

In order to propagate the wave from a source to various coastal locations, we use embedded grids, placing a coarse grid in deep water and coupling it with finer grids in shallow water areas. We use an interactive grid splicing: the equations are solved in all grids at each time step, and the values along the boundaries are interpolated every time step. Figure 2 shows graphically how this interaction works. Capital and small letters in the figure are used to denote coarse and fine grid values, respectively. First, velocities and sea level are computed over the coarse and fine grids. Coarse grid values, which overlap the fine grid, are updated with fine grid values. Then, values around the edge of the fine grid, which are needed for computations within the fine grid, are linearly interpolated between the coarse and fine grids. Circled sea levels and velocities in the figure represent estimated points used in the fine grid calculations. As an example of how the interpolation is made, points 1 and 2 are given by:

$$u(j5,k10) = (U(J8,K7) + 2U(J9,K7))/3$$
$$\xi(j1,k6) = (8\xi(J7,K6) + 4\xi(J7,K5) + 9\xi(j2,k4) + 2\xi(J6,K6) + \xi(J6,K5))/24$$

This algorithm describes any space decrease from the coarse to the fine grid, if expressed by an odd whole number (1:3, 1:5, 1:7, …).

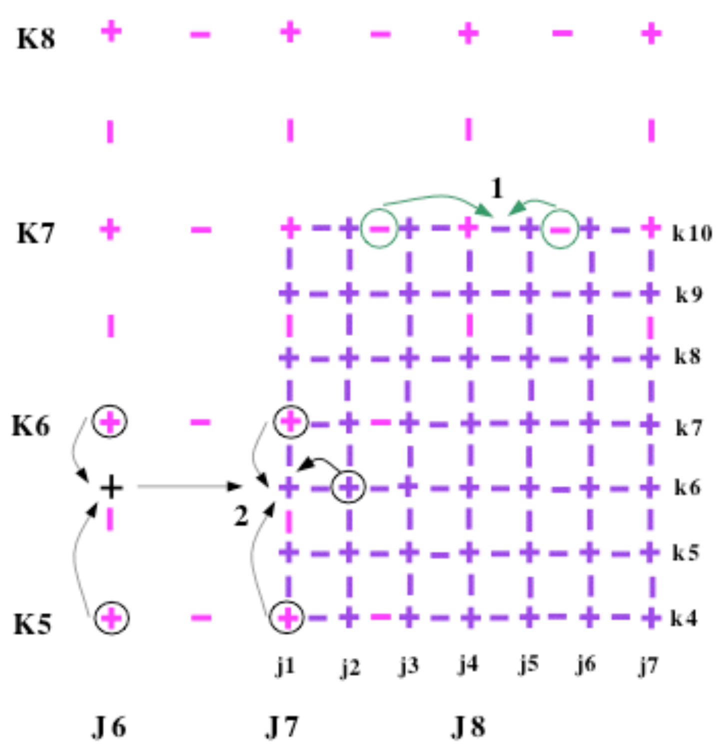

Figure 2. Grid splicing technique.